ex. 1 $f(x) = x^4 + 6x^3 + 12x^2 + 8x + 1$ → y-int = 1

$f'(x) = 4x^3 + 18x^2 + 24x + 8$

$f''(x) = 12x^2 + 36x + 24$

x-int = −1, −.16

$f'(x)$: − 0 − 0 + ; ↘ −2 ↘ $-\frac{1}{2}$ ↗ ; min

$f''(x)$: + 0 − 0 + ; ∪ −2 infl. pt ∩ −1 infl. pt ∪

$4x^3 + 18x^2 + 24x + 8 = 0$

$2x^3 + 9x^2 + 12x + 4 = 0$

crit #'s $x = -\frac{1}{2}, -2$

$12x^2 + 36x + 24 = 0$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

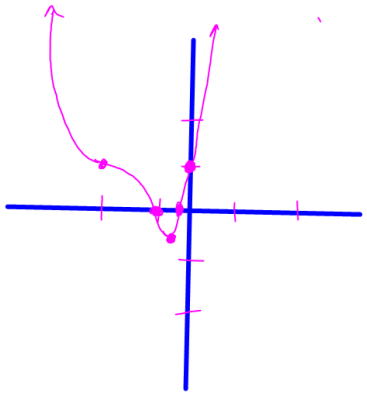
$x = -1, -2$

$f(0) = 1$ y-int

$f(-2) = 1$

$f(-\frac{1}{2}) = -\frac{11}{16}$

$f(-1) = 0$



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ex 2 $f(x) = \frac{x^2 - 3}{x^3}$, $x \neq 0$

$\lim_{x \to 0^-} f(x) = \infty$

$\lim_{x \to 0^+} f(x) = -\infty$

$f'(x) = \frac{2x(x^3) - (x^2-3)(3x^2)}{x^6}$; $2x^4 - 3x^4 + 9x^2$

$f'(x) = \frac{-x^4 + 9x^2}{x^6} = \frac{-x^2 + 9}{x^4}$ ⇒ crit #'s $x = \pm 3$ / $x \neq 0$

$f''(x) = \frac{-2x(x^2) - (-x^2+9)(4x)}{x^8}$; $-2x^3$; $+4x^3 - 36x$

$= \frac{2x^2 - 36}{x^5}$ ⇒ crit #'s $\pm\sqrt{18}$ / $x \neq 0$

$f(x) = \frac{x^2 - 3}{x^3}$

intercepts

no y-int

x-int = $\pm\sqrt{3}$

horiz

$y = 0$

horiz

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

x-int

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$f(-3) = \frac{6}{-27} = -\frac{2}{9}$

$f(3) = \frac{6}{27} = \frac{2}{9}$

$f(-\sqrt{18}) = -.196$

$f(\sqrt{18}) = .196$

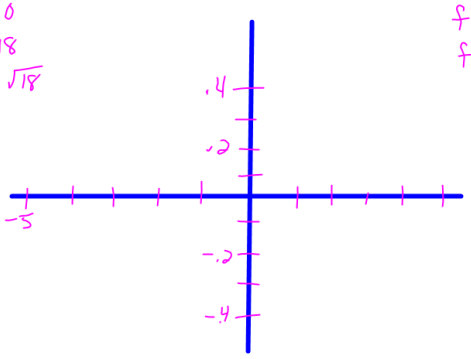
$-2x^2 - 36 = 0$

$x^2 = 18$

$x = \pm\sqrt{18}$

$f'(x)$: − 0 + / + 0 − ; ↘ −3 ↗ 0 ↗ 3 ↘ ; min ; max

$f''(x)$: − 0 + / − 0 + ; ∩ $-\sqrt{18}$ ∪ 0 ∩ $\sqrt{18}$ ∪



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