ex.1

$f(x) = x^4 + 6x^3 + 12x^2 + 8x + 1 \rightarrow$ y-int $= 1$

x-int $= -1, -.16$

$f'(x) = 4x^3 + 18x^2 + 24x + 8$

$f''(x) = 12x^2 + 36x + 24$

$f'(x)$: $-$ 0 $-$ 0 $+$; $\searrow$ -2 $\searrow$ $-\frac{1}{2}$ $\nearrow$ min

$f''(x)$: $+$ 0 $-$ 0 $+$; $\cup$ -2 infl. pt $\cap$ -1 infl. pt $\cup$

$4x^3 + 18x^2 + 24x + 8 = 0$

$2x^3 + 9x^2 + 12x + 4 = 0$

crit #'s

$x = -\frac{1}{2}, -2$

$12x^2 + 36x + 24 = 0$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

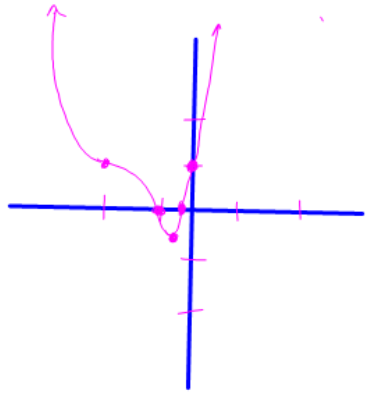
$x = -1, -2$

$f(0) = 1$ y-int

$f(-2) = 1$

$f(-\frac{1}{2}) = -\frac{11}{16}$

$f(-1) = 0$



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ex.2

$f(x) = \frac{x^2-3}{x^3}, \; x \neq 0$

$\lim_{x \to 0^-} f(x) = \infty$

$\lim_{x \to 0^+} f(x) = -\infty$

$f'(x) = \frac{\overset{2x^4}{\cancel{2x(x^3)}} - \overset{-3x^4+9x^2}{(x^2-3)(3x^2)}}{x^6}$

$f'(x) = \frac{-x^4+9x^2}{x^6} = \frac{-x^2+9}{x^4} \Rightarrow$ crit #'s $x = \pm 3$ / $x \neq 0$

$f''(x) = \frac{\overset{-2x^2}{-2x(x^2)} - \overset{+4x^2-36}{(-x^2+9)(4x^3)}}{x^8} = \frac{2x^2-36}{x^5} \Rightarrow$ crit #'s $\pm\sqrt{18}$ / $x \neq 0$

$-2x^2 - 36 = 0$

$x^2 = 18$

$x = \pm\sqrt{18}$

$f(x) = \frac{x^2-3}{x^3}$

horiz

$y = 0$

horiz

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to -\infty} f(x) = 0$

Intercepts

no y-int

x-int $= \pm\sqrt{3}$

x-int

$x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$f(-3) = \frac{6}{-27} = -\frac{2}{9}$

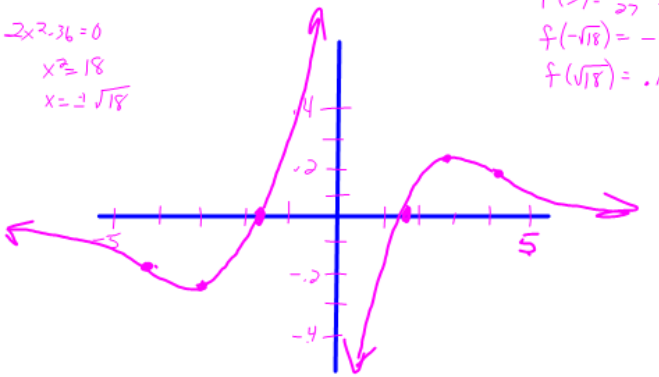
$f(3) = \frac{6}{27} = \frac{2}{9}$

$f(-\sqrt{18}) = -.196$

$f(\sqrt{18}) = .196$

$f'(x)$: $-$ 0 $+$ $\boxed{x}$ $+$ 0 $-$; $\searrow$ -3 $\nearrow$ 0 $\nearrow$ 3 $\searrow$; min, max

$f''(x)$: $-$ 0 $+$ $\boxed{x}$ $-$ 0 $+$; $\cap$ $-\sqrt{18}$ $\cup$ 0 $\cap$ $\sqrt{18}$ $\cup$



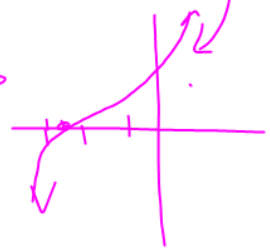
p.306

1, 3, 5

ex. 4 $f(x) = \dfrac{1}{x^3+3x^2+3x+3}$, $x \neq -2.6$ $\quad x^3+3x^2+3x+3=0$

$\lim_{x \to -2.26^-} f(x) = -\infty$

$\lim_{x \to -2.26^+} f(x) = \infty$



$f'(x) = \dfrac{0-(3x^2+6x+3)}{(x^3+3x^2+3x+3)^2}$

$f'(x) = \dfrac{-3x^2-6x-3}{(x^3+3x^2+3x+3)^2}$, crit #: -1 / $x \neq -2.26$

$-3x^2-6x-3=0$

$-3(x^2+2x+1)=0$

$-3(x+1)^2=0$

$-6(x+1)$

$f''(x) = \dfrac{(-6x-6)(x^3+3x^2+3x+3)^{\cancel{2}} - (-3x^2-6x-3)(2)\cancel{(x^3+3x^2+3x+3)}(3x^2+6x+3)}{(x^3+3x^2+3x+3)^{\cancel{4}\,3}}$

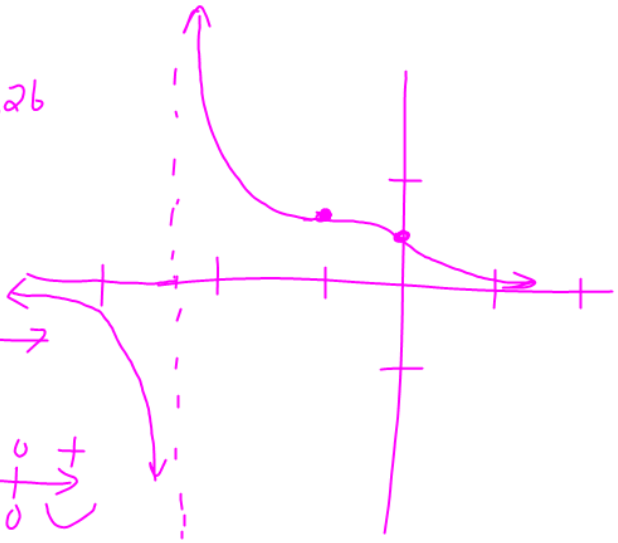
$3(x+1)^2$

crit #'s: $0, -1$ / $x \neq -2.26$

$f(x) = \dfrac{1}{x^3+3x^2+3x+3}$

$f(-1) = \frac{1}{2}$

$f(0) = \frac{1}{3}$



$f'(x)$: $-$ $\boxed{x}$ $-$ 0 $-$; -2.26, -1

$f''(x)$: $-$ $\boxed{x}$ $+$ 0 $-$ 0 $+$; -2.26, -1, 0

y-int	x-int	hori
$\frac{1}{3}$	none	$y=0$

vert

$x = -2.26$

p. 306			
1, 3, 5, 11, 15, 21, 29			