㉑ $f(x) = \dfrac{x^2+1}{3x^2-1}$, $x \neq \pm\sqrt{\dfrac{1}{3}}$

$f'(x) = \dfrac{2x(3x^2-1) - (x^2+1)(6x)}{(3x^2-1)^2}$

$f'(x) = \dfrac{-8x}{(3x^2-1)^2}$ $\rightarrow$ Crit #
$x=0$

$f''(x) = \dfrac{-8(3x^2-1)^2 + (+8x)(2)(3x^2-1)(6x)}{(3x^2-1)^{\times 3}}$

(with annotations: $-24x^2+8$, $+96x^2$)

$f''(x) = \dfrac{72x^2+8}{(3x^2-1)^3}$

$3x^2-1=0$
$x^2 = \dfrac{1}{3}$
$x = \pm\sqrt{\dfrac{1}{3}} = \pm\dfrac{\sqrt{3}}{3}$

$\lim\limits_{x \to -\sqrt{\frac{1}{3}}^-} f(x) = $
$\lim\limits_{x \to -\sqrt{\frac{1}{3}}^+} f(x) = $ } vert. asymptote

$\lim\limits_{x \to \sqrt{\frac{1}{3}}^-} f(x)$
$\lim\limits_{x \to \sqrt{\frac{1}{3}}^+} f(x)$

$f(x) = \dfrac{x^2+1}{3x^2-1}$

horiz
$y = \dfrac{1}{3}$

Intercepts
no x-int
y-int = -1

$f(0) = -1$

$f'(x)$: $\xleftarrow{\quad + \;\boxed{x}\; + \; 0 \; - \;\boxed{x}\; - \quad}$
$-\sqrt{\frac{1}{3}}$ $\nearrow$ 0 max $\sqrt{\frac{1}{3}}$ $\searrow$

$f''(x)$: $\xleftarrow{\quad + \;\boxed{x}\; - \;\boxed{x}\; + \quad}$
$\cup$ $-\sqrt{\frac{1}{3}}$ $\cap$ $\sqrt{\frac{1}{3}}$ $\cup$
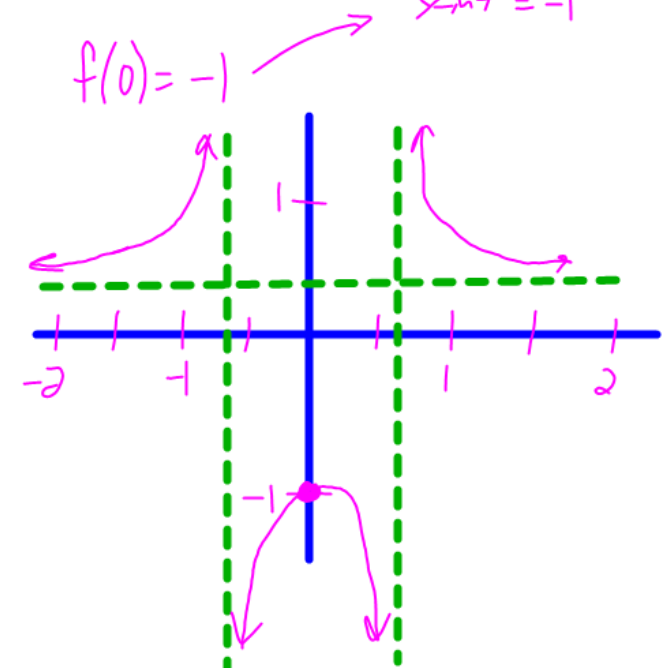
(25) $f(x) = x^4 - 16x^3 + 42x^2 - 39.6x + 14$

$f'(x) = 4x^3 - 48x^2 + 84x - 39.6 \rightarrow$ <u>crit #'s</u>

<u>X = .895, 1.11, 9.999</u>

$f''(x) = 12x^2 - 96x + 84$
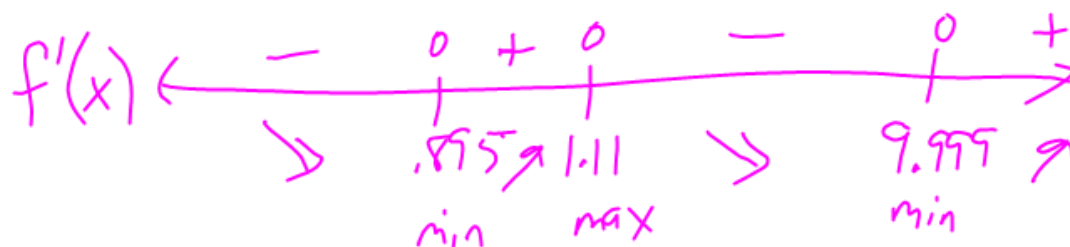
$12(x^2 - 8x + 7)$

$12(x-7)(x-1)$

<u>crit #'s</u>
$x = 1, 7$

<u>intercepts</u>

y-int = 14

x-int = 1.52
12.9

$f'(x)$ ← ——— $\underset{.895}{\overset{-}{|}}$ + $\underset{1.11}{\overset{0}{|}}$ — $\underset{9.999}{\overset{0}{|}}$ + →

↘ .895 ↗ 1.11 ↘ 9.999 ↗
min   max   min

$f(.895) = 1.372$
$f(1.11) = 1.428$
$f(9.999) = -2182$
$f(1) = 1.4$
$f(7) = -1292$

$f''(x)$ ← + $\underset{}{\overset{0}{|}}$ — $\underset{}{\overset{0}{|}}$ + →

⌣ ⌢ ⌣
infl!   infl.
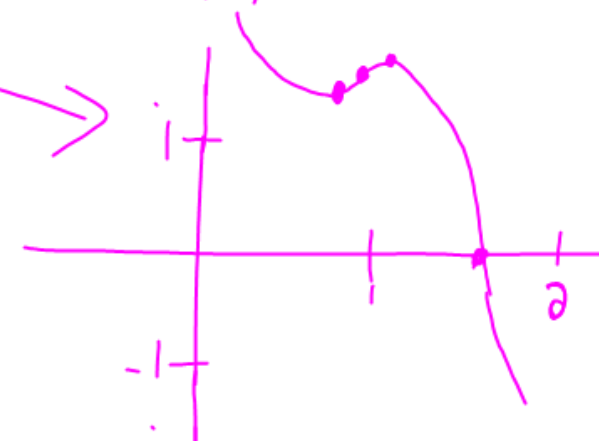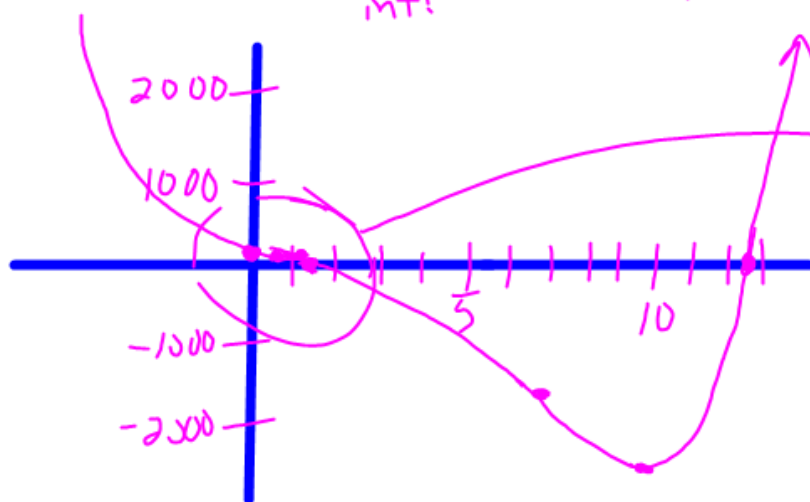
(11) $f(x) = \dfrac{4x}{x^2 - x + 1}$

$\dfrac{1 \mp \sqrt{(-1)^2 - 4(1)(1)}}{2(1)} = \dfrac{1 \pm \boxed{(-5)}}{2}$

$f'(x) = \dfrac{\overset{4x^2 - 4x + 4}{4(x^2 - x + 1)} - \overset{-8x^2 + 4x}{4x(2x - 1)}}{(x^2 - x + 1)^2}$

$-4x^2 + 4 = 0$

$x^2 = 1$

$x = \pm 1$

$f'(x) = \dfrac{-4x^2 + 4}{(x^2 - x + 1)^2}$, $\quad \begin{array}{c} \text{crit #'s} \\ x = 1, -1 \end{array}$

$f'(x) \xleftarrow{\quad - \quad \overset{0}{\underset{-1}{\big|}} \quad + \quad \overset{0}{\underset{}{\big|}} \quad - \quad} \rightarrow$

$\searrow \quad \underset{min}{-1} \quad \nearrow \quad \underset{max}{\big|} \quad \searrow$

$f''(x) = \dfrac{(-8x)(x^2 - x + 1)^{\cancel{2}} \ominus (-4x^2 + 4)\overset{(4x-2)}{(2)}\cancel{(x^2 - x + 1)}(2x - 1)}{(x^2 - x + 1)^{*3}}$

$\dfrac{-8x^3 + 8x^2 - 8x + 16x^3 - 8x^2 - 16x + 8}{(x^2 - x + 1)^3}$

$f''(x) = \dfrac{8x^3 - 24x + 8}{(x^2 - x + 1)^3} = \dfrac{8(x^3 - 3x + 1)}{(x^2 - x + 1)^3}$