1. $\dfrac{\cot A}{\tan A} = \dfrac{\frac{\cos A}{\sin A}}{\frac{\sin A}{\cos A}} = \dfrac{\cos A}{\sin A} \cdot \dfrac{\cos A}{\sin A} = \dfrac{\cos^2 A}{\sin^2 A} = \cot^2 A$

$= \dfrac{\cot A}{\frac{1}{\cot A}} = \cot A \cdot \dfrac{\cot A}{1} = \cot^2 A$

2. $\cos x + \sin x \tan x$

$\cos x + \sin x \dfrac{\sin x}{\cos x} \dfrac{\cos x}{\cos x} \left(\dfrac{\cos x}{1} + \dfrac{\sin^2 x}{\cos x}\right.$

$\dfrac{\cos^2 x}{\cos x} + \dfrac{\sin^2 x}{\cos x} = \dfrac{\cos^2 x + \sin^2 x}{\cos x} = \dfrac{1}{\cos x} = \sec x$

3. $\sin^2\theta\cos^2\theta - \cos^2\theta$

$\cos^2\theta(\sin^2\theta - 1)$

$\cos^2\theta(-\cos^2\theta)$

$-\cos^4\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\sin^2\theta - 1 = -\cos^2\theta$

$(1-\cos^2\theta)\cos^2\theta - \cos^2\theta$

$\cos^2\theta - \cos^4\theta - \cos^2\theta$

$-\cos^4\theta$

4. $(\sin x + \cos x)^2 + (\sin x - \cos x)^2$

$(\sin x + \cos x)(\sin x + \cos x)$

$\sin^2 x + 2\sin x\cos x + \cos^2 x + \sin^2 x - 2\sin x\cos x + \cos^2 x$

$1 + 1$

2

$2\sin^2 x + 2\cos^2 x$

$2(\sin^2 x + \cos^2 x)$

$2 \cdot 1$

5. $(1+\cos\theta)(\csc\theta-\cot\theta)$

$(1+\cos\theta)\left(\frac{1}{\sin\theta}-\frac{\cos\theta}{\sin\theta}\right)$

$\left(\frac{1+\cos\theta}{1}\right)\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$\frac{1-\cos^2\theta}{\sin\theta}$

$\frac{\sin^2\theta}{\sin\theta}$

$\sin\theta$

$\csc\theta-\cot\theta+\cos\theta\csc\theta-\cos\theta\cot\theta$

$\frac{1}{\sin\theta}-\cot\theta+\cos\theta\frac{1}{\sin\theta}-\cos\theta\frac{\cos\theta}{\sin\theta}$

$+\frac{\cos\theta}{\sin}$

$\frac{1}{\sin\theta}-\cancel{\cot\theta}+\cancel{\cot\theta}-\frac{\cos^2\theta}{\sin\theta}$

$\frac{1-\cos^2\theta}{\sin\theta}$

$$\frac{1-\cos x}{1-\cos x}\frac{\sin x}{1+\cos x}+\frac{\sin x}{1-\cos x}\frac{1+\cos x}{1+\cos x} \Big/ \sin x\left(\frac{1}{1+\cos x}+\frac{1}{1-\cos x}\right)$$

$$\frac{\sin x-\sin x\cos x}{1-\cos^2 x}+\frac{\sin x+\sin x\cos x}{1-\cos^2 x}$$

$$\frac{\sin x-\cancel{\sin x\cos x}+\sin x+\cancel{\sin x\cos x}}{1-\cos^2 x}$$

$$\frac{2\sin x}{\sin^2 x}$$

$$\frac{2}{\sin x}=2\left(\frac{1}{\sin x}\right)$$

$$2\csc x$$

6. $\cos^4\theta + 2\cos^2\theta \sin^2\theta \equiv \sin^4\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta(\cos^2\theta + 2\sin^2\theta) - \sin^4\theta$

$(1-\sin^2\theta)(1-\sin^2\theta+2\sin^2\theta) - \sin^4\theta$

$(1-\sin^2\theta)(1+\sin^2\theta) - \sin^4\theta$

$1 - \sin^4\theta - \sin^4\theta$

$1 - 2\sin^4\theta$

$\sin^2\theta \sin^2\theta$

$(1-\cos^2\theta)(1-\cos^2\theta)$

$-(1-2\cos^2\theta+\cos^4\theta)$

x^2+2x+1

$(x+1)(x+1)$

$(x+1)^2$

$x^4+2x^2+1^4$

$(x^2+1)(x^2+1)$

6. $\cos^4\theta + 2\cos^2\theta\sin^2\theta + \sin^4\theta$

$(\cos^2\theta+\sin^2\theta)(\cos^2\theta+\sin^2\theta)$

$1 \cdot 1$

1

$(\cos^2\theta+\sin^2\theta)^2$

1^2