23. sec 1275°

$\cos 1275° = \cos 195° = \cos(150+45)$

$= \cos 150° \cos 45 - \sin 150 \sin 45$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\cos 1275° = \frac{-\sqrt{6} - \sqrt{2}}{4}$

$\sec 1275° = \frac{4}{-\sqrt{6}-\sqrt{2}} \cdot \left(\frac{-\sqrt{6}+\sqrt{2}}{-\sqrt{6}+\sqrt{2}}\right) = \frac{4(\ )}{6-2}$

$= \frac{\cancel{4}(-\sqrt{6}+\sqrt{2})}{\cancel{4}}$

$= -\sqrt{6}+\sqrt{2}$

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21 $\tan \frac{23\pi}{12} = \tan 345° = \tan(300+45)$

$= \frac{\tan 300 + \tan 45}{1 - \tan 300 \tan 45} = \frac{-\sqrt{3}+1}{1-(-\sqrt{3})(1)}$

$= \frac{-\sqrt{3}+1}{1+\sqrt{3}}\left(\frac{1-\sqrt{3}}{1-\sqrt{3}}\right) = \frac{-\sqrt{3}+3+1-\sqrt{3}}{1-3}$

$= \frac{4-2\sqrt{3}}{-2} = -2+\sqrt{3}$

39. $\sin(A + B) = \dfrac{\tan A + \tan B}{\sec A \sec B}$

$$\sin A \cos B + \cos A \sin B = \frac{\tan A}{\sec A \sec B} + \frac{\tan B}{\sec A \sec B}$$

$$= \frac{\dfrac{\sin A}{\cos A}}{\dfrac{1}{\cos A \cos B}} + \frac{\dfrac{\sin B}{\cos B}}{\dfrac{1}{\cos A \cos B}}$$

$$= \frac{\sin A}{\cancel{\cos A}} \cdot \frac{\cancel{\cos A} \cos B}{1} + \frac{\sin B}{\cancel{\cos B}} \cdot \frac{\cos A \cancel{\cos B}}{1}$$

$$\sin A \cos B + \cos A \sin B = \sin A \cos B + \cos A \sin B$$

35. $\cos(60° + A) = \sin(30° - A)$

$$\cos 60 \cos A - \sin 60 \sin A = \sin 30 \cos A - \cos 30 \sin A$$

$$\frac{1}{2} \cos A - \frac{\sqrt{3}}{2} \sin A = \frac{1}{2} \cos A - \frac{\sqrt{3}}{2} \sin A$$

24. $\csc \frac{5\pi}{12} = \csc 75^\circ$

$\sin 75^\circ = \sin(45+30) = \sin 45 \cos 30 + \cos 45 \sin 30$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$\sin \frac{5\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\csc \frac{5\pi}{12} = \frac{4}{\sqrt{6}+\sqrt{2}} \cdot \left(\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}\right) = \frac{\quad}{6-2}$

$= \frac{\cancel{4}(\sqrt{6}-\sqrt{2})}{\cancel{4}}$

$\csc \frac{5\pi}{12} = \sqrt{6}-\sqrt{2}$

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(36) $\sin(A+\pi) = -\sin A$

$\sin A \cos \pi + \cos A \sin \pi =$

$\sin A(-1) + \cancel{\cos A(0)} =$

$-\sin A = -\sin A$

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(42) $\sin(x+y)\sin(x-y) = \sin^2 x - \sin^2 y$

$(\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$

$\sin^2 x \cos^2 y - \cos^2 x \sin^2 y =$

$\sin^2 x(1-\sin^2 y) - (1-\sin^2 x)\sin^2 y$

$\sin^2 x - \cancel{\sin^2 x \sin^2 y} - \sin^2 y + \cancel{\sin^2 x \sin^2 y}$

$\sin^2 x - \sin^2 y = \sin^2 x - \sin^2 y$

$0° < x < 90°$

① If $\sin x = \frac{2}{3}$, find $\cos x$.

② If $\tan x = \frac{7}{2}$, find $\sin x$.