① $\sin x = \frac{2}{3}$, $\cos x$

$\left(\frac{2}{3}\right)^2 + \cos^2 x = 1 - \frac{4}{9}$

$-\frac{4}{9}$

$\cos^2 x = \frac{5}{9}$

$\cos x = \frac{\sqrt{5}}{3}$

---

② $\tan x = \frac{7}{2}$, $\sin x$

$\cot x = \frac{2}{7}$

$1 + \left(\frac{2}{7}\right)^2 = \csc^2 x$

$\sin x = \frac{7}{\sqrt{53}}$ ⟵ $1 + \frac{4}{49} =$

$\boxed{\sin x = \frac{7\sqrt{53}}{53}}$ $\frac{53}{49} = \csc^2 x$

$\frac{\sqrt{53}}{7} = \csc x$

---

③ Find the value of sin $(x - y)$ if $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, $\sin x = \frac{9}{41}$, and sin $y = \frac{7}{25}$.

$\sin(x - y) = \sin x \cos y - \cos x \sin y$

$= \left(\frac{9}{41}\right)\left(\frac{24}{25}\right) - \left(\frac{40}{41}\right)\left(\frac{7}{25}\right)$

$= \frac{216}{1025} - \frac{280}{1025}$

$= \boxed{-\frac{64}{1025}}$

$\sin^2 x + \cos^2 x = 1$

$\left(\frac{9}{41}\right)^2 + \cos^2 x = 1$

$-\frac{81}{1681} \qquad -\frac{81}{1681}$

$\cos^2 x = \frac{1600}{1681}$

$\cos x = \frac{40}{41}$

$\left(\frac{7}{25}\right)^2 + \cos^2 y = 1$

$-\frac{49}{625} \qquad -\frac{49}{625}$

$\cos^2 y = \frac{576}{625}$

$\cos y = \frac{24}{25}$

Find
$\cos(x+y)$ $\sin x = \frac{4}{9}$ and $\sin y = \frac{1}{4}$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

$= \left(\frac{\sqrt{65}}{9}\right)\left(\frac{\sqrt{15}}{4}\right) - \left(\frac{4}{9}\right)\left(\frac{1}{4}\right)$

$= \frac{\sqrt{975} - 4}{36}$ $\frac{\sqrt{975}}{\sqrt{25}\sqrt{39}}$

$= \frac{5\sqrt{39} - 4}{36}$

$\left(\frac{4}{9}\right)^2 + \cos^2 x = 1$

$-\frac{36}{81} \qquad \frac{-36}{81}$

$\cos^2 x = \frac{65}{81}$

$\cos x = \frac{\sqrt{65}}{9}$

$\left(\frac{1}{4}\right)^2 + \cos^2 y = 1$

$-\frac{1}{16} \qquad -\frac{1}{16}$

$\cos^2 y = \frac{15}{16}$

$\cos y = \frac{\sqrt{15}}{16}$

---

9. $\tan(x+y)$ if $\csc x = \frac{5}{3}$ and $\cos y = \frac{5}{13}$

$\sec y = \frac{13}{5}$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$= \frac{\frac{15}{20}\cdot\frac{3}{4} + \frac{1}{5}\cdot\frac{48}{20}}{1 - \left(\frac{3}{4}\right)\left(\frac{12}{5}\right)\frac{36}{20}}$

$= \frac{\frac{63}{20}}{-\frac{16}{20}} = \frac{63}{20} \cdot \frac{20}{16} = \boxed{-\frac{63}{16}}$

$1 + \cot^2 x = \left(\frac{5}{3}\right)^2 - 1$
$-1 \qquad \frac{25}{9}$

$\cot^2 x = \frac{16}{9}$

$\cot x = \frac{4}{3}$

$\tan x = \frac{3}{4}$

$\tan^2 y + 1 = \left(\frac{13}{5}\right)^2 - 1$
$-1 \qquad \frac{169}{25}$

$\tan^2 y = \frac{144}{25}$

$\tan y = \frac{12}{5}$

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Quiz Monday

7.1-7.3