

$$(13) \quad f(x) = \frac{x^2 - 5x + 4}{x}, \quad x \neq 0$$

$$f'(x) = \frac{(2x-5)x - (x^2-5x+4)}{x^2} = \frac{x^2-4}{x^2} \quad \begin{array}{l} \text{crit \#s} \\ 2, -2 \end{array}$$

$$f''(x) = \frac{2x(x^2) - (x^2-4)(2x)}{x^4} = \frac{8x}{x^4} \quad f'(x) = 0$$

$$f''(2) = \frac{8(2)}{2^4} \rightarrow \text{pos.} \quad x=2 \text{ min}$$

$$f''(-2) = \frac{8(-2)}{(-2)^4} \rightarrow \text{neg} \quad x=-2 \text{ max}$$

$$(15) \quad f(x) = (x^2+1)^{2/3}$$

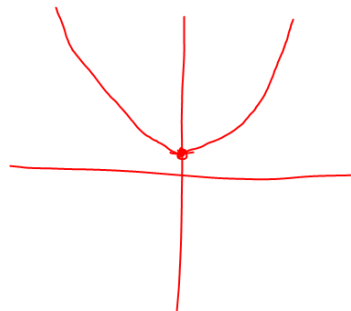
$$f'(x) = \frac{2}{3}(x^2+1)^{-1/3}(2x) = \frac{4x}{3(x^2+1)^{1/3}} \quad \begin{array}{l} \text{crit \#s} \\ x=0 \end{array}$$

$$f''(x) = \frac{4}{3}(x^2+1)^{-1/3} + \frac{4x}{3}(-\frac{1}{3})(x^2+1)^{-4/3}(2x)$$

$$\frac{3(x^2+1)}{3(x^2+1)^{1/3}} - \frac{8x^2}{9(x^2+1)^{4/3}}$$

$$f''(x) = \frac{12x^2+12-8x^2}{9(x^2+1)^{4/3}} = \frac{4x^2+12}{9(x^2+1)^{4/3}} \Rightarrow \text{always concave up}$$

$$f(0) = 1$$



$$f'(x) \begin{array}{c} - \quad + \\ \leftarrow \quad \rightarrow \\ \text{min} \end{array}$$

② $f(x) = x^{3/4} - 4x^{1/4} = \sqrt[4]{x^3} - 4\sqrt[4]{x}, x \geq 0$

$f'(x) = \frac{3}{4}x^{-1/4} - x^{-3/4} = \frac{x^{3/4}}{4x^{1/4}} - \frac{1}{x^{3/4}} \cdot \frac{1}{4}$

$f'(x) = \frac{3\sqrt{x} - 4}{4x^{3/4}}$ crit #'s $3\sqrt{x} - 4 = 0$
 $x = 0, \frac{16}{9}$
 $\sqrt{x} = \frac{4}{3}$
 $x = \frac{16}{9}$

$f''(x) = -\frac{3}{16}x^{-5/4} + \frac{3}{4}x^{-7/4} = \frac{x^{1/4}}{16x^{5/4}} - \frac{3}{4x^{7/4}} \cdot \frac{1}{4}$

$f''(x) = \frac{-3\sqrt{x} + 12}{16x^{7/4}}$, crit #'s $x = 0, 16$
 $-3\sqrt{x} + 12 = 0$
 $\sqrt{x} = 4$
 $x = 16$

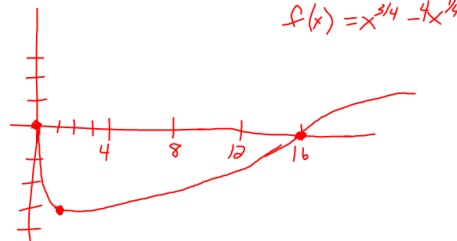
$f'(x)$ $\leftarrow \begin{array}{c} \boxed{-} \\ 0 \end{array} \begin{array}{c} \boxed{+} \\ \frac{16}{9} \end{array} \rightarrow$

$f(0) = 0$

$f(\frac{16}{9}) = -3.1$

$f''(x)$ $\leftarrow \begin{array}{c} \boxed{+} \\ 0 \end{array} \begin{array}{c} \boxed{-} \\ 16 \end{array} \rightarrow$

$f(16) = 0$



③ $f(x) = \frac{x^2}{x^2 - 9}, x \neq \pm 3$ horiz asymptote $y = 1$

$\lim_{x \rightarrow 3^+} = \infty$

$\lim_{x \rightarrow 3^-} = -\infty$

$f'(x) = \frac{-18x}{(x^2 - 9)^2} \Rightarrow$ crit #'s $x = 0$

$\lim_{x \rightarrow 3^+} = -\infty$

$f''(x) = \frac{-18(x^2 - 9)^{-2} + (-18x)(-2)(x^2 - 9)^{-3}(2x)}{(x^2 - 9)^4} = \frac{54x^2 + 162}{(x^2 - 9)^3}$

$\lim_{x \rightarrow 3^-} = \infty$

$f'(x)$ $\leftarrow \begin{array}{c} \boxed{+} \\ -3 \end{array} \begin{array}{c} \boxed{+} \\ 0 \end{array} \begin{array}{c} \boxed{-} \\ 3 \end{array} \rightarrow$

$f(0) = 0$

$f''(x)$ $\leftarrow \begin{array}{c} \boxed{+} \\ -3 \end{array} \begin{array}{c} \boxed{-} \\ 0 \end{array} \begin{array}{c} \boxed{+} \\ 3 \end{array} \rightarrow$

