

$$\begin{aligned}
 19. \tan 22.5^\circ &= \tan \frac{45^\circ}{2} = \sqrt{\frac{1 - \cos 45^\circ}{1 + \cos 45^\circ}} \\
 &= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{1 + \frac{\sqrt{2}}{2}}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{\frac{2+\sqrt{2}}{2}}} \Rightarrow \frac{2-\sqrt{2}}{2} \cdot \frac{2}{2+\sqrt{2}} \\
 &= \sqrt{\frac{\frac{2-\sqrt{2}}{2} \cdot \frac{2-\sqrt{2}}{2}}{4-2}} = \sqrt{\frac{4-2\sqrt{2}-2\sqrt{2}+2}{4-2}} = \sqrt{\frac{6-4\sqrt{2}}{2}} \\
 &\quad \boxed{\sqrt{3-2\sqrt{2}}} \\
 &\quad \sqrt{\frac{(2-\sqrt{2})^2}{4-2}} = \frac{\sqrt{(2-\sqrt{2})^2}}{\sqrt{2}} = \frac{2-\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \\
 &\quad \frac{2\sqrt{2}-2}{2} \neq \boxed{\sqrt{2}-1}
 \end{aligned}$$

$$\begin{aligned}
 30. (\sin \theta + \cos \theta)^2 - 1 &= \sin 2\theta \\
 (\sin \theta + \cos \theta)(\sin \theta + \cos \theta) & \\
 \sin^2 \theta + 2 \sin \theta \cos \theta + \cos^2 \theta - 1 &= \\
 1 + 2 \sin \theta \cos \theta - 1 &= \\
 2 \sin \theta \cos \theta &= 2 \sin \theta \cos \theta
 \end{aligned}$$

$$28. \csc 2\theta = \frac{1}{2} \sec \theta \csc \theta$$

$$\frac{1}{\sin 2\theta} = \frac{1}{2} \cdot \frac{1}{\cos \theta} \cdot \frac{1}{\sin \theta}$$

$$\frac{1}{2\sin \theta \cos \theta} = \frac{1}{2\sin \theta \cos \theta}$$

$$32. \sec 2\theta = \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta - \sin^2 \theta}$$

$$\frac{1}{\cos 2\theta} = \frac{1}{\cos^2 \theta - \sin^2 \theta}$$

$$\frac{1}{\cos^2 \theta - \sin^2 \theta} = \frac{1}{\cos^2 \theta - \sin^2 \theta}$$

$$\sin x = \frac{1}{2}$$

60° ref.

$$\tan \theta = -\sqrt{3}$$

$$x = 30^\circ, 150^\circ$$

$$\theta = 120^\circ, 300^\circ$$