19. $\tan 22.5^\circ = \tan \frac{45^\circ}{2} = \sqrt{\frac{1-\cos 45}{1+\cos 45}}$

$$= \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{\frac{2+\sqrt{2}}{2}}}$$

$$\frac{2-\sqrt{2}}{\cancel{2}} \cdot \frac{\cancel{2}}{2+\sqrt{2}}$$

$$= \sqrt{\frac{2-\sqrt{2}}{2+\sqrt{2}} \cdot \frac{2-\sqrt{2}}{2-\sqrt{2}}} = \sqrt{\frac{4-2\sqrt{2}-2\sqrt{2}+2}{4-2}}$$

$$= \sqrt{\frac{6-4\sqrt{2}}{2}} = \sqrt{3-2\sqrt{2}}$$

$$= \sqrt{\frac{(2-\sqrt{2})^2}{4-2}} = \frac{2-\sqrt{2}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{2\sqrt{2}-2}{2} = \sqrt{2}-1$$

21. $\cos\theta = \frac{4}{5}, 0° < \theta < 90°$

$\sin\theta = \frac{3}{5}$

$\tan\theta = \frac{3}{4}$

$\sin^2\theta + \left(\frac{4}{5}\right)^2 = 1$

$\sin^2\theta = \frac{9}{25}$

$\sin\theta = \frac{3}{5}$

$\tan\theta = \frac{3/5}{4/5} = \frac{3}{\cancel{5}} \cdot \frac{\cancel{5}}{4} = \frac{3}{4}$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$\sin 2\theta = \frac{24}{25}$

$\cos 2\theta = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$\cos 2\theta = \frac{7}{25}$

$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2}$ $\frac{9}{16}$

$= \frac{\frac{6}{4}}{\frac{7}{16}} = \frac{6}{\cancel{4}_1} \cdot \frac{\cancel{16}^4}{7}$

$= \frac{24}{7}$

22. $\sin\theta = \frac{1}{3}, 0 < \theta < \frac{\pi}{2}$

$\left(\frac{1}{3}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{8}{9}$

$\cos\theta = \frac{2\sqrt{2}}{3}$

$\tan\theta = \frac{1/3}{2\sqrt{2}/3} = \frac{1}{\cancel{3}} \cdot \frac{\cancel{3}}{2\sqrt{2}} =$

$\frac{1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4} = \tan\theta$

$\sin 2\theta = 2\left(\frac{1}{3}\right)\left(\frac{2\sqrt{2}}{3}\right)$

$= \frac{4\sqrt{2}}{9}$

$\cos 2\theta = 2\left(\frac{2\sqrt{2}}{3}\right)^2 - 1$

$= 2\left(\frac{8}{9}\right) - 1$

$= \frac{16}{9} - \frac{9}{9}$

$= \frac{7}{9}$

$\tan 2\theta = \frac{{}^1\cancel{2}\left(\frac{\sqrt{2}}{\cancel{4}_2}\right)}{1-\left(\frac{\sqrt{2}}{4}\right)^2}$

$= \frac{\frac{\sqrt{2}}{\cancel{2}}}{\frac{7}{\cancel{8}}} = \frac{\sqrt{2}}{\cancel{2}} \cdot \frac{\cancel{8}^4}{7} = \frac{4\sqrt{2}}{7}$

28. $\csc 2\theta = \frac{1}{2}\sec\theta\csc\theta$

$$\frac{1}{\sin 2\theta} = \frac{1}{2}\cdot\frac{1}{\cos\theta}\cdot\frac{1}{\sin\theta}$$

$$\frac{1}{2\sin\theta\cos\theta} = \frac{1}{2\sin\theta\cos\theta}$$

30. $(\sin\theta + \cos\theta)^2 - 1 = \sin 2\theta$

$$(\sin\theta+\cos\theta)(\sin\theta+\cos\theta) - 1$$

$$\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta - 1 =$$

$$2\sin\theta\cos\theta + 1 - 1 =$$

$$2\sin\theta\cos\theta = 2\sin\theta\cos\theta$$

---

32. $\sec 2\theta = \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta}$

$$\frac{1}{\cos 2\theta} = \frac{1}{\cos^2\theta - \sin^2\theta}$$

$$\frac{1}{\cos 2\theta} = \frac{1}{\cos 2\theta}$$