31. $4\sin^2 x + 1 = -4\sin x$ $\quad 0 \le x < 2\pi$

$4\sin^2 x + 4\sin x + 1 = 0$

$(2\sin x + 1)(2\sin x + 1) = 0$

$2\sin x + 1 = 0$

$\sin x = -\frac{1}{2}$

$x = 210^\circ, 330^\circ$

$x = \frac{7\pi}{6}, \frac{11\pi}{6}$

32. $\sqrt{2}\tan x = 2\sin x$

$\sqrt{2}\tan x - 2\sin x = 0$

$\sqrt{2}\frac{\sin x}{\cos x} - 2\sin x = 0$

$\sin x\left(\frac{\sqrt{2}}{\cos x} - 2\right) = 0$

$\sin x = 0$

$x = 0^\circ, 180^\circ$

$\frac{\sqrt{2}}{\cos x} - 2 = 0$

$\frac{\sqrt{2}}{\cos x} = 2$

$\sqrt{2} = 2\cos x$

$\frac{\sqrt{2}}{2} = \cos x$

$x = 45^\circ, 315^\circ$

$x = 0, \pi, \frac{\pi}{4}, \frac{7\pi}{4}$

38. $\cos x \tan x - 2\cos^2 x = -1$

$\cos x \frac{\sin x}{\cos x}$

$\sin x - 2\cos^2 x = -1$

$\sin x - 2(1-\sin^2 x) = -1$

$\sin x - 2 + 2\sin^2 x = -1$
$\quad +1 \qquad\qquad +1$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$ $\qquad$ $\sin x + 1 = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\sin x = -1$

$x = 30°, 150°$ $\qquad$ $x = 270°$

$$\left.\begin{aligned} x &= \frac{\pi}{6} + 2\pi k \\ x &= \frac{5\pi}{6} + 2\pi k \\ x &= \frac{3\pi}{2} + 2\pi k \end{aligned}\right\} k = \text{integer}$$

for all real

$\tan = \frac{y}{x} = \frac{\sin}{\cos}$

39. $3\tan^2 x = \sqrt{3}\tan x$

$3\tan^2 x - \sqrt{3}\tan x = 0$

$\tan x(3\tan x - \sqrt{3}) = 0$

$\tan x = 0$ $\qquad$ $3\tan x - \sqrt{3} = 0$

$x = 0°, 180°$ $\qquad$ $\tan x = \frac{\sqrt{3}}{3}$

$x = 0, \pi, \frac{\pi}{6}, \frac{2\pi}{6}$ $\qquad$ $x = 30°, 210°$

$$\left.\begin{aligned} x &= \pi k \\ x &= \frac{\pi}{6} + \pi k \end{aligned}\right\} k = \text{integer}$$

$0 \le x < 2\pi$

34. $\cot^2 x - \csc x = 1$

$\csc^2 x - 1 - \csc x = 1$

$\csc^2 x - \csc x - 2 = 0$

$(\csc x + 1)(\csc x - 2) = 0$

$\csc x + 1 = 0$ $\qquad$ $\csc x - 2 = 0$

$\csc x = -1$ $\qquad$ $\csc x = 2$

$\sin x = -1$ $\qquad$ $\sin x = \frac{1}{2}$

$x = 270^\circ$ $\qquad$ $x = 30^\circ, 150^\circ$

$x = \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$

33. $\sin x = \cos 2x - 1$

$0 = \cos 2x - \sin x - 1$

$0 = 1 - 2\sin^2 x - \sin x - 1$

$0 = -2\sin^2 x - \sin x$

$0 = \sin x(-2\sin x - 1)$

$\sin x = 0$ $\quad$ $-2\sin x - 1 = 0$

$x = 0°, 180°$ $\quad$ $\sin x = -\frac{1}{2}$

$x = 210°, 330°$

$x = 0, \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

46. $\sec^2 x + 2\sec x = 0$

$\sec x(\sec x + 2) = 0$

$\cancel{\sec x = 0}$ $\quad$ $\sec x + 2 = 0$

$\cancel{\cos x \to \text{und.}}$ $\quad$ $\sec x = -2$

$\cos x = -\frac{1}{2}$

$x = 120°, 240°$

$\left.\begin{array}{l} x = \frac{2\pi}{3} + 2\pi k \\ x = \frac{4\pi}{3} + 2\pi k \end{array}\right\}$ k = integer

30 $\quad (2\sin x - 1)(2\cos^2 x - 1) = 0$

$2\sin x - 1 = 0$ $\quad$ $2\cos^2 x - 1 = 0$

$\sin x = \frac{1}{2}$ $\quad$ $\sqrt{\cos^2 x} = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{1}}{\sqrt{2}}$

$x = 30, 150°$ $\quad$ $\cos x = \pm\frac{\sqrt{2}}{2}$

$x = 45°, 315°, 135, 225$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$