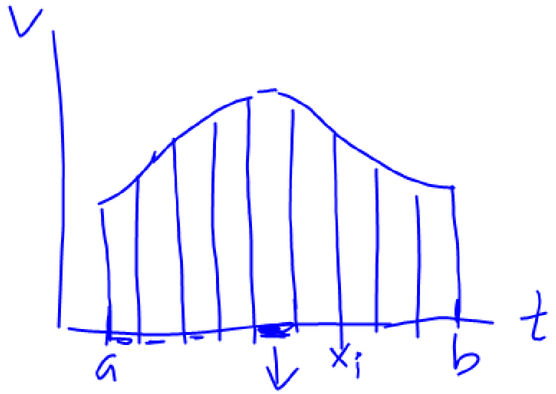


$[a, b]$

$$\frac{b-a}{n} = \Delta x$$

$$X_0 = a$$

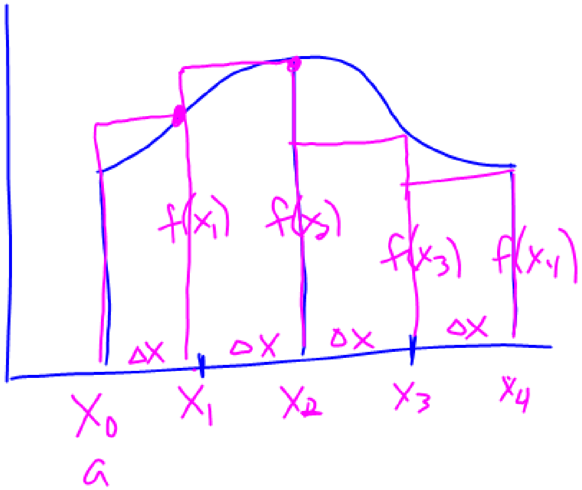
$$X_1 = X_0 + \Delta x$$

$$X_2 = \cancel{x_1} + \Delta x = X_0 + \Delta x(2)$$

$$X_3 = \qquad = x_0 + \Delta x(3)$$

$$\longleftrightarrow \quad \boxed{X_i = x_0 + \Delta x\, i}$$

right endpoint estimation



$$A_{rea} = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$

$$= \Delta x\left[f(x_1) + f(x_2) + f(x_3) + f(x_4)\right]$$

$$A_n = \sum_{i=1}^{n} f(x_i)\,\Delta x \qquad \boxed{3.1}$$

ex. 1 $\quad y = f(x) = 2x - 2x^2 \qquad [0, 1]$ (a, b) $\qquad 10 = n$ rectangles

$$A_{10} = \sum_{i=1}^{10} f(x_i)\,\Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{1-0}{10} = \frac{1}{10}$$

$$x_i = x_0 + \Delta x\, i = 0 + \frac{1}{10} i$$

$$x_i = \frac{1}{10} i = \frac{i}{10}$$

$$f\left(\tfrac{1}{10} i\right) = 2\left(\tfrac{i}{10}\right) - 2\left(\tfrac{i}{10}\right)^2$$

$$= \sum_{i=1}^{10} f\left(\tfrac{1}{10} i\right) \frac{1}{10}$$

$$= \sum_{i=1}^{10} \left[\frac{i}{5} - \frac{i^2}{50}\right] \frac{1}{10}$$

$$= \frac{1}{50} \sum_{i=1}^{10} i - \frac{1}{500} \sum_{i=1}^{10} i^2$$

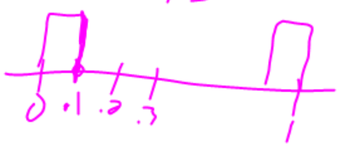
$$= \frac{1}{50}\left(\frac{10(11)}{2}\right) - \frac{1}{500}\left(\frac{10(11)(21)}{6}\right) = \frac{33}{100} = .33$$

$$f(x) = 2x - 2x^2$$

$$A_{10} = \sum_{i=1}^{10} f(x_i)\,\Delta x$$

$$\Delta x = \frac{1}{10} \qquad [0, 1]$$

$$= \underbrace{\left[f(.1) + f(.2) + f(.3) + \dots + f(1)\right]}_{\sum_{i=1}^{4} f(x_i)} \Delta x$$



$$= .18 + .32$$

ex. 2

$$A_{20} = \sum_{i=1}^{20} f(x_i)\,\Delta x \qquad n=20 \qquad \Delta x = \frac{1-0}{20} = \frac{1}{20}$$

$$x_i = 0 + \frac{1}{20} i \qquad x_i = \frac{i}{20}$$

$$f(x) = 2x - 2x^2$$

$$= \sum_{i=1}^{20} \left[2\left(\frac{i}{20}\right) - \cancel{2}\left(\frac{i}{20}\right)^2 \right] \frac{1}{20} \qquad \frac{i^2}{\cancel{400}\ 200}$$

$$= \frac{1}{200} \sum_{i=1}^{20} i - \frac{1}{4000} \sum_{i=1}^{20} i^2$$

$$= \frac{1}{200}\left(\frac{20(21)}{2}\right) - \frac{1}{4000}\left(\frac{20(21)(41)}{6}\right)$$

$$= .3325$$

Defn. 3.1

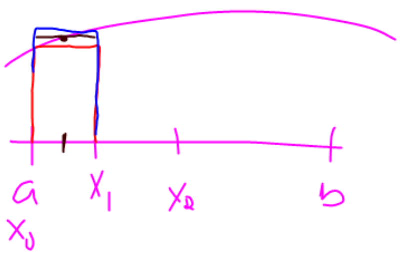
$$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

---

left

right

midpoint

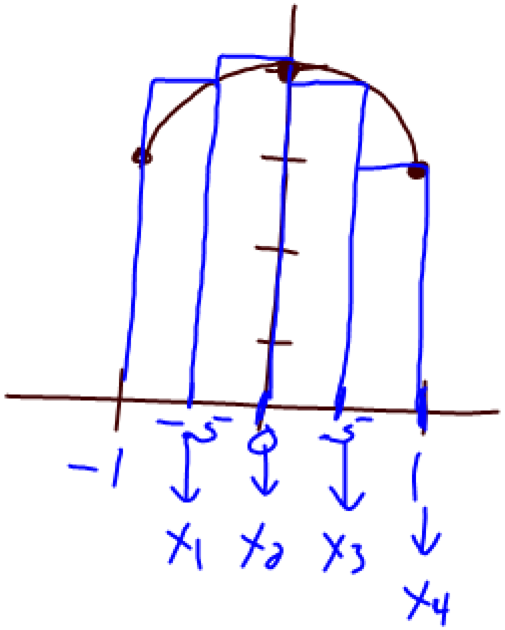


right : $x_i = a + \Delta x\, i$

left : $x_i = a + \Delta x (i-1)$

midpoint : $x_i = a + \Delta x \left(i - \frac{1}{2}\right)$

$f(x) = 4 - x^2$ right $[-1, 1]$ $n = 4$



$\Delta x = \dfrac{1-(-1)}{4} = \dfrac{1}{2}$

$$A_4 = \underbrace{\sum_{i=1}^{4} f(x_i)\,\Delta x}_{\downarrow}$$

$x_i = -1 + \frac{1}{2} i$

$x_i = -1 + \frac{i}{2}$

$$= [f(x_1) + f(x_2) + f(x_3) + f(x_4)]\,\Delta x$$

$$= [f(-.5) + f(0) + f(.5) + f(1)]\,\Delta x$$

$$= (3.75 + 4 + 3.75 + 3)\,\frac{1}{2}$$

$$=$$