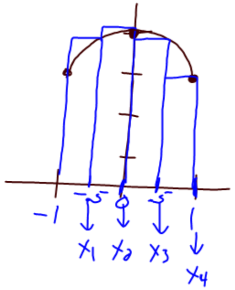
$f(x) = 4 - x^2$ right $[-1, 1]$ $n = 4$

$\Delta x = \frac{1-(-1)}{4} = \frac{1}{2}$



$$A_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x$$

$\cancel{x_i = -1 + \tfrac{1}{2}i}$

$\cancel{x_i = -1 + \tfrac{i}{2}}$

$$= [f(x_1) + f(x_2) + f(x_3) + f(x_4)]\,\Delta x$$

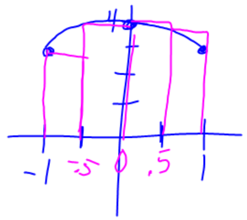
$$= [f(-.5) + f(0) + f(.5) + f(1)]\,\Delta x$$

$$= (3.75 + 4 + 3.75 + 3)\,\tfrac{1}{2}$$

$$= 7.25$$

$f(x) = 4 - x^2$ left $[-1, 1]$ ($a = -1$, $b = 1$) $n = 4$

$\Delta x = \frac{1-(-1)}{4} = \frac{1}{2}$



$$A_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x$$

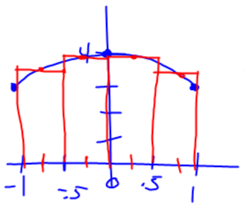
$$= [f(-1) + f(-.5) + f(0) + f(.5)]\,\Delta x$$

$$= [3 + 3.75 + 4 + 3.75)\,\tfrac{1}{2}$$

$$= 7.25$$

$f(x) = 4 - x^2$ midpoint $\Delta x = \frac{1}{2}$

$[-1, 1]$



$$A_4 = \sum_{i=1}^{4} f(x_i)\,\Delta x$$

$$= [f(-.75) + f(-.25) + f(.25) + f(.75)]\,\Delta x$$

$$= (3.4375 + 3.9375 + 3.9375 + 3.4375)\,\tfrac{1}{2}$$

$$= 7.375$$

$$A = \lim_{n\to\infty} A_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

ex.3 $f(x) = 2x - 2x^2$ $[0,1]$ $n\to\infty$

$\Delta x = \frac{1-0}{n} = \frac{1}{n}$

right
$x_i = a + \Delta x\, i$

$x_i = 0 + \frac{1}{n} i = \frac{i}{n}$

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \sum_{i=1}^{n} \left[2\left(\frac{i}{n}\right) - 2\left(\frac{i}{n}\right)^2\right]\frac{1}{n}$$

$$= \sum_{i=1}^{n}\left(\frac{2i}{n^2} - \frac{2i^2}{n^3}\right) = \frac{2}{n^2}\sum_{i=1}^{n} i - \frac{2}{n^3}\sum_{i=1}^{n} i^2$$

$$= \frac{\cancel{2}}{n^{\cancel{2}}}\left(\frac{\cancel{n}(n+1)}{\cancel{2}}\right) - \frac{\cancel{2}}{n^{\cancel{3}}}\left(\frac{\cancel{n}(n+1)(2n+1)}{\cancel{6}\,3}\right)$$

$$= \frac{n+1}{n} - \frac{2n^2+3n+1}{3n^2} \Longrightarrow \frac{3n^2+3n}{3n^2} - \frac{2n^2+3n+1}{3n^2}$$

$$= \frac{n}{n} + \frac{1}{n} - \frac{2n^2}{3n^2} - \frac{3n}{3n^2} - \frac{1}{3n^2} \qquad = \frac{n^2-1}{3n^2} = \frac{(n+1)(n-1)}{3n^2}$$

$$A = \lim_{n\to\infty}\left(1 + \cancel{\frac{1}{n}}^{\to 0} - \frac{2}{3} - \cancel{\frac{1}{n}}^{\to 0} - \cancel{\frac{1}{3n^2}}^{\to 0}\right)$$

$$= 1 - \frac{2}{3}$$

$$= \frac{1}{3}$$

ex.4 $y = f(x) = \sqrt{x+1}$ $[1,3]$ $n\to\infty$

$\Delta x = \frac{3-1}{n} = \frac{2}{n}$

$x_i = 1 + \frac{2}{n} i$

$$A_n = \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \sum_{i=1}^{n}\left(\sqrt{1+\frac{2i}{n}+1}\right)\frac{2}{n} = \frac{2}{n}\sum_{i=1}^{n}\sqrt{\frac{2i}{n}+2}$$

p. 367-369

1-27 odd