

5. Simplify $\cos A \tan^2 A + \cos A$.

$$\cancel{\cos A} \frac{\sin^2 A}{\cancel{\cos A}} + \cos A$$

$$\frac{\sin^2 A}{\cos A} + \frac{\cos A}{1} \cdot \frac{\cos A}{\cos A}$$

$$\frac{\sin^2 A + \cos^2 A}{\cos A}$$

$$\frac{1}{\cos A}$$

sec A

$$\cos A (\sec^2 A - 1) + \cos A$$

$$\cos A \left(\frac{1}{\cos^2 A} - 1 \right) + \cos A$$

$$\frac{1}{\cos A} - \frac{\cos A}{\cos A} + \frac{\cos A}{\cos A}$$

$$\frac{1}{\cos A}$$

sec A

28. $\sec x \csc x = \tan x + \cot x$

$$\frac{1}{\cos x} \cdot \frac{1}{\sin x} = \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x}$$

$$= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} = \frac{1}{\sin x \cos x}$$

$$\begin{aligned}
 31. \quad 1 + \sin 2x &= (\sin x + \cos x)^2 \\
 &\quad (\sin x + \cos x)(\sin x + \cos x) \\
 &= \underline{\sin^2 x} + \underline{\sin x \cos x} + \underline{\sin x \cos x} + \underline{\cos^2 x} \\
 &= \underline{\sin^2 x + \cos^2 x} + \underline{2 \sin x \cos x} \\
 &= 1 + \sin 2x
 \end{aligned}$$

$$\begin{aligned}
 16. \quad \cos \frac{5\pi}{8} &= \cos 112.5^\circ = \cos \frac{225^\circ}{2} \\
 &= -\sqrt{\frac{1 + \cos 225^\circ}{2}} = -\sqrt{\frac{\frac{2}{2} + (-\frac{\sqrt{2}}{2})}{2}} \\
 &= -\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = -\sqrt{\frac{2-\sqrt{2}}{4}} \\
 &= -\frac{\sqrt{2-\sqrt{2}}}{2}
 \end{aligned}$$

8. If $\cos^2 x + 2 \sin x - 2 = 0$, find the exact value of $\sin x$

$$1 - \sin^2 x + 2 \sin x - 2 = 0$$

$$-1 (-\sin^2 x + 2 \sin x - 1 = 0)$$

$$\sin^2 x - 2 \sin x + 1 = 0$$

$$(\sin x - 1)(\sin x - 1) = 0$$

$$\sin x - 1 = 0$$

$$\sin x = 1$$

22. $2 \sin^2 x = -\sqrt{3} \sin x$ for $0 \leq x < 2\pi$ ^{radians}

$$2 \sin^2 x + \sqrt{3} \sin x = 0$$

$$\sin x (2 \sin x + \sqrt{3}) = 0$$

$$\sin x = 0 \quad 2 \sin x + \sqrt{3} = 0$$

$$x = 0^\circ, 180^\circ$$

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$x = 240^\circ, 300^\circ$$

$$x = 0, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$$

$$18. \sin x = \frac{3}{5}, 0 < x < \frac{\pi}{2}$$

$$\sin 2x = 2 \sin x \cos x$$

$$= 2 \left(\frac{3}{5} \right) \left(\frac{4}{5} \right)$$

$$\sin 2x = \frac{24}{25}$$

$$\cos 2x = \left(\frac{4}{5} \right)^2 - \left(\frac{3}{5} \right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$\cos 2x = \frac{7}{25}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$= \frac{2 \left(\frac{3}{4} \right)}{\frac{16}{16} - \left(\frac{3}{4} \right)^2} = \frac{\frac{6}{4}}{\frac{7}{16}} = \frac{6}{4} \cdot \frac{16}{7}$$

$$\tan 2x = \frac{24}{7}$$

$$\left(\frac{3}{5} \right)^2 + \cos^2 x = 1$$

$$\cos^2 x = \frac{16}{25}$$

$$\cos x = \frac{4}{5}$$

$$\tan x = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}$$

$$\tan = \frac{3}{4}$$