$$\cos 2\theta = \cos(\underset{\alpha}{\theta} + \underset{\beta}{\theta})$$

$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$

$$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$

$$\cos 2\theta = \underline{\cos^2\theta} - \sin^2\theta$$

$$= 1 - \sin^2\theta - \sin^2\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \cos^2\theta - (1 - \cos^2\theta)$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$\boxed{\cos 2\theta = 2\cos^2\theta - 1}$$

$$\sin 2\theta = \sin(\theta + \theta)$$

$$= \sin\theta\cos\theta + \cos\theta\sin\theta$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

$$\tan 2\theta = \tan(\theta + \theta)$$

$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$$

$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}}$$

If $\sin\theta = \frac{2}{3}$. $\quad 0° < \theta < 90°$

Find

1a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right)$

$\sin 2\theta = \frac{4\sqrt{5}}{9}$

$\left(\frac{2}{3}\right)^2 + \cos^2\theta = 1$

$\cos^2\theta = \frac{5}{9}$

$\cos\theta = \frac{\sqrt{5}}{3}$

1b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{\sqrt{5}}{3}\right)^2 - \left(\frac{2}{3}\right)^2$

$= \frac{1}{9}$

$= 2\cos^2\theta - 1$

$= 2\left(\frac{\sqrt{5}}{3}\right)^2 - 1$

$= 2\left(\frac{5}{9}\right) - 1$

$= \frac{10}{9} - \frac{9}{9}$

$= \frac{1}{9}$

$1 - 2\sin^2\theta$

$1 - 2\left(\frac{2}{3}\right)^2$

$1 - 2\left(\frac{4}{9}\right)$

$\frac{9}{9} - \frac{8}{9}$

$\frac{1}{9}$

$\sin\theta = \frac{2}{3} \qquad \cos\theta = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{\frac{2}{3}}{\frac{\sqrt{5}}{3}}$

$= \frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{\sqrt{5}}$

$= \frac{2}{\sqrt{5}}$

$\tan\theta = \frac{2\sqrt{5}}{5}$

$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$

$= \frac{2\left(\frac{2\sqrt{5}}{5}\right)}{1 - \left(\frac{2\sqrt{5}}{5}\right)^2}$

$= \frac{\frac{4\sqrt{5}}{5}}{1 - \frac{\cancel{20}}{\cancel{25}}\,\frac{4}{5}}$

$= \frac{\frac{4\sqrt{5}}{5}}{\frac{1}{5}} = \frac{4\sqrt{5}}{\cancel{5}} \cdot \frac{\cancel{5}}{1} = 4\sqrt{5}$

$\alpha = 210^\circ$

$$\sin\frac{7\pi}{12} = \sin 105^\circ = \sin\frac{210^\circ}{2}$$

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\sin\frac{210}{2} = \sqrt{\frac{1-\cos 210^\circ}{2}} = \sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}}$$

$$= \sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2+\sqrt{3}}{4}} = \frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

$\alpha = 135^\circ$

2b. $\cos 67.5^\circ = \cos\frac{135^\circ}{2}$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\cos\frac{135^\circ}{2} = \sqrt{\frac{1+\cos 135^\circ}{2}}$$

$$= \sqrt{\frac{1+\left(-\frac{\sqrt{2}}{2}\right)}{2}} = \sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{4}}$$

$$= \frac{\sqrt{2-\sqrt{2}}}{2}$$

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