$$
\begin{aligned}
& \cos 2 \theta=\cos (\theta+\theta) \\
& \cos 2 \theta=\cos ^{2} \theta-\sin ^{2} \theta \\
& \sin ^{2} \theta+\cos ^{2} \theta=1 \\
& =\cos \theta \cos \theta-\sin \theta \sin \theta \\
& \cos 2 \theta=\cos ^{2} \theta-\sin ^{2} \theta \\
& \sin 2 \theta=\sin (\theta+\theta) \\
& =\sin \theta \cos \theta+\cos \theta \sin \theta \\
& \sin 2 \theta=2 \sin \theta \cos \theta \\
& \cos 2 \theta=\cos ^{2} \theta-\underline{\operatorname{shn}^{2} \theta} \\
& =\cos ^{2} \theta-\left(1-\cos ^{2} \theta\right) \\
& =\cos ^{2} \theta-1+\cos ^{2} \theta \\
& \cos 2 \theta=2 \cos ^{2} \theta-1 \\
& \tan 2 \theta=\tan (\theta+\theta) \\
& =\frac{\tan \theta+\tan \theta}{1-\tan \theta \tan \theta} \\
& \tan 2 \theta=\frac{2 \tan \theta}{1-\tan ^{2} \theta}
\end{aligned}
$$

$\sin \theta=\frac{2}{3} \quad 0^{\circ}<\theta<90^{\circ}$
(1a)

$$
\begin{aligned}
\sin 2 \theta & =2 \sin \theta \cos \theta \\
& =2\left(\frac{2}{3}\right)\left(\frac{\sqrt{5}}{3}\right) \\
\sin 2 \theta & =\frac{4 \sqrt{5}}{9}
\end{aligned}
$$

$$
\left(\frac{2}{3}\right)^{2}+\cos ^{2} \theta=1
$$

$$
\cos ^{2} \theta=\frac{5}{9}
$$

$$
\cos \theta=\frac{\sqrt{5}}{3}
$$

(b)

$$
\sin \theta=\frac{2}{3}
$$

$$
\begin{aligned}
\cos 2 \theta & =1-2 \sin ^{2} \theta
\end{aligned}=\cos ^{2} \theta-\sin ^{2} \theta=2 \cos ^{2} \theta-1.10=\left(\frac{2}{3}\right)^{2}-\left(\frac{\sqrt{5}}{3}\right)^{2}=2 \cdot\left(\frac{\sqrt{5}}{3}\right)^{2}-1 \text { } \begin{aligned}
& =1-2\left(\frac{10}{9}-1\right. \\
& =1-\frac{8}{9} \\
\cos 2 \theta & \left.=\frac{1}{9}\right) \\
& =\frac{5}{9}-\frac{4}{9} \\
& =\frac{1}{9}
\end{aligned}
$$

(1.)

$$
\begin{aligned}
& \tan 2 \theta=\frac{2 \tan \theta}{1-\tan ^{2} \theta} \\
&=\frac{2\left(\frac{2 \sqrt{5}}{5}\right)}{1-\left(\frac{2 \sqrt{5}}{5}\right)^{2}} \cdot \frac{264}{25} 5 \tan \theta=\frac{2}{\sqrt{5}}=\frac{2 \sqrt{\sqrt{5}}}{5} \\
&=\frac{2}{\sin \theta}=\frac{2}{3} \cos \theta=\frac{\sqrt{5}}{3} \\
& \tan \theta=\frac{2}{3} \\
&=\frac{\frac{4 \sqrt{5}}{5}}{5}=\frac{4 \sqrt{5}}{5} \cdot \frac{87}{1}=4 \sqrt{5}
\end{aligned}
$$

$$
\begin{aligned}
& \text { a. } \begin{aligned}
& \sin \frac{7 \pi}{12}=\sin 105^{\circ}=\sin \frac{210^{\circ}}{2} \\
& \begin{aligned}
\sin \frac{\alpha}{2} & =\sqrt{\frac{1-\cos \alpha}{2}} \\
\sin \frac{210}{2} & =\sqrt{\frac{1-\cos 210^{\circ}}{2}}=\sqrt{\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{2}} \\
& =\sqrt{\frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{2}}=\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2}{1}}}=\sqrt{\frac{2+\sqrt{3}}{4}}=\frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}}
\end{aligned}=\frac{\sqrt{2+\sqrt{3}}}{2} \\
& \cos 67,5^{\circ}=\cos \frac{135^{\circ}}{2} \\
& \cos \frac{135^{\circ}}{2}=\sqrt{\frac{1+\cos 135^{\circ}}{2}}=\sqrt{\frac{\frac{2}{2}-\frac{\sqrt{2}}{2}}{2}} \\
&=\sqrt{\frac{\frac{2-\sqrt{2}}{2}}{2}}=\sqrt{\frac{2-\sqrt{2}}{4}}=\frac{\sqrt{2-\sqrt{2}}}{2}
\end{aligned}
\end{aligned}
$$



